Survey of Viscous Interactions Associated with High Mach Number Flight

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I. Introduction

THE most severe problems of atmospheric flight at high Mach numbers are associated with viscous-inviscid interactions. Cruise vehicles for Mach numbers above four and lifting re-entry vehicles have highly complex three-dimensional configurations in which exist many regions of high compression that can cause boundary layers to separate. Although separation can result in loss of control effectiveness or flow degradation in an engine inlet, flow reattachment gives rise to heat rates that can far exceed those for an attached boundary layer. A further, and possibly far more severe viscous interaction problem is the impingement of shock waves generated by the forebody and other external components of a vehicle on aft sections resulting in local heat rates that may be many times larger than stagnation point values.

Peak heating conditions may be laminar for lifting re-entry configurations, though our knowledge of boundary layer-transition is far from adequate so that transitional and turbulent flows cannot be ruled out. However, Reynolds numbers of potential high Mach number cruise vehicles are high—10^8 to 10^9—so that viscous interactions will be predominately associated with turbulent boundary layers and their attendant higher heat rates.

The high local heat rates resulting from viscous interactions cause "hot spots" that could lead to catastrophic failure. Vivid examples of damage resulting from viscous interactions are given in Figs. 1 and 2. A ventral pylon on the X-15 airplane, shown in Fig. 1, caused high local heating of the fuselage around its root, and developed large holes near its tip due to the impingement of the shock wave from a dummy ramjet it supported, during a flight at Mach 6.7 in 1967. A study of the flowfield of the pylon-mounted dummy ramjet configuration is reported in Ref. 1. Figure 2 shows considerable damage due to interaction heating to the underside of a sled and its supporting slipper as a result of a run at 7000 fps on the 7-mile test track at Holloman Air Force Base.

Unlike stagnation-point heating where the location is obvious, the problem with complex configurations is to determine "where" high heat rates are likely to occur, as well as their magnitude.

It is the purpose of this paper to identify some potentially critical areas of viscous interactions associated with high Mach number vehicles and briefly review our state of knowledge in these areas with emphasis on the basic flow phenomena.

II. Viscous Interaction Problem Areas

Typical areas of strong viscous interactions on a hypothetical high Mach number cruise vehicle are illustrated in Fig. 3.

A deflected flap or a shock-boundary layer interaction in an inlet will generally cause flow separation with high heat rates at reattachment in addition to possible degradation of the flow entering a combustion chamber. Such flows may be subject to spanwise nonuniformities of the oncoming stream as well as finite span effects. In order to develop prediction methods for such flows, the comparatively simpler problem of two-dimensional boundary-layer separation must be well understood, and is discussed in the section on compression corners and shock interactions.

Other interactions are mostly associated with three-dimensional flows for which even the inviscid flowfield is not generally well known. The interaction of the bow shock of a fin with the boundary layer on a wing can cause widespread separation and vorticity of a highly three-dimensional nature. In fact, any external protuberance on a vehicle, whether large or small—of the order of the boundary-layer thickness—is a possible source of local separation and high heating. Axial corners formed by compression surfaces, such as in air breathing engine inlets, at wing-body and fin-wing junctions, and in turbomachinery, give rise to complex flow patterns with vortices and embedded shocks which can interact with surface boundary layers and result in zones of high heating as well as flow degradation. Finally, the fuselage bow shock may impinge on a wing, and a wing shock may impinge on a fin, potentially causing extremely high heat rates locally at leading edges.

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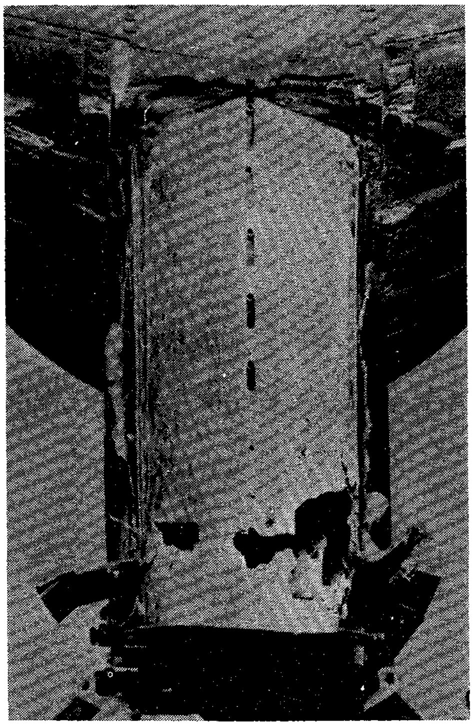
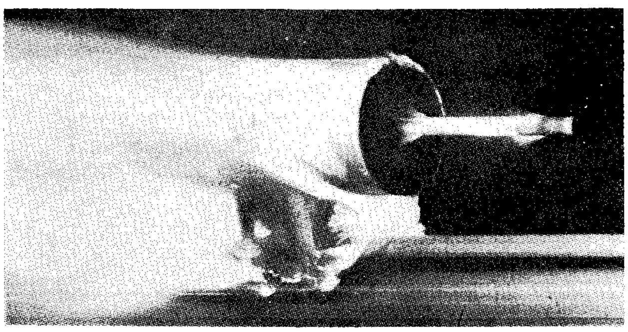
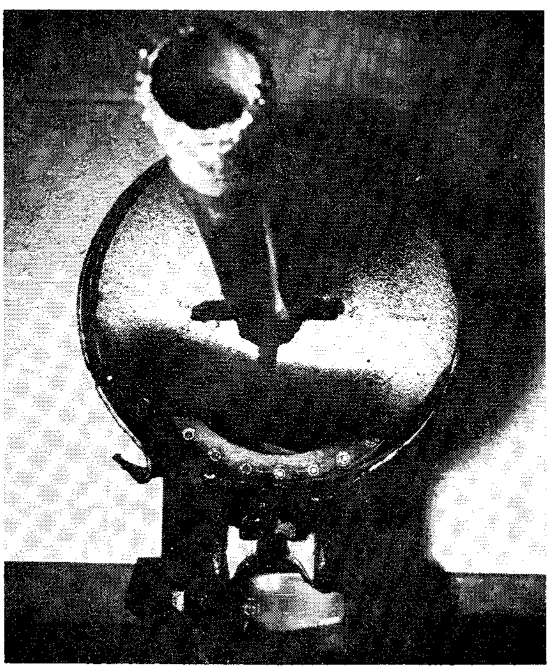


Fig. 1 X-15 ventral pylon damaged by interaction heating (courtesy NASA).



a) In motion at 7000 fps



b) Resulting interaction heating damage

Fig. 2 Sled on test track at Holloman Air Force Base (courtesy AFMDC, U.S. Air Force)

Vortex flows such as may arise on the lee side of a fuselage or wing-body at angle of attack, causing possible regions of high heating and interference with control surfaces, have been well covered by Rainbird[2] and, therefore, are not specifically included in this review. However, strong vortical flows are discussed in their connection with the three-dimensional interactions considered herein. Also, as emphasis is placed on high heating due to flow reattachment on a vehicle surface, the hypersonic leading edge interaction, base flows and wakes, and boundary layers on curved surfaces are not included except as related to the interactions discussed.

The classification of three-dimensional viscous interactions in the following sections—fin interaction, axial corner flow, shock impingement—is based largely on the representative flow models which have been studied. There are clearly some features of similarity between these flow types. For example, one can associate with the fin interaction problem both shock impingement and corner flow at the fin-wing junction. However, the section on fin interaction will deal primarily with the strong shock generated by a blunt leading edge; whereas, the section on corner flow concentrates on the interaction resulting from two adjacent sharp edged compression surfaces, and the section on shock impingement deals with the interaction of externally generated strong shock with leading edges.

Three-dimensional interactions are discussed somewhat more extensively than two-dimensional ones, as they are generally not as well known, but nevertheless represent the vast majority of practical viscous interaction problems. Among general studies of viscous interactions are the extensive investigations of flow separation related to hypersonic control surface configurations of Kaufman et al.[3] and the more recent work reported by Bertram and Henderson[4] on hypersonic interactions on flaps, in corners, and on wing body combinations. Studies of two- and three-dimensional shock-boundary-layer interactions associated with hypersonic controls are also given in Refs. 5 and 6. A monumental summary of viscous interaction literature with a bibliography of over 900 entries is given by Ryan.[7]

With the broad spectrum of viscous interaction problems covered, and particularly the considerable volume of work on two-dimensional flows, only a small cross section of the pertinent literature is presented herein. Apologies are offered for errors of omission.

III. Compression Corners and Shock Interactions

The problem of flow separation in compression corners and due to shock-boundary-layer interaction has been the object of a large number of investigations over the past two decades. One of the earliest studies is Liepmann's[8] notable work on the interaction of shock waves with the boundary layer on circular arc profiles in transonic flow which shows not only the perturbation to the flowfield, but also the considerable difference between the laminar and turbulent interactions. A comprehensive bibliography of works up to 1966 on flow separation at high speeds is given in Ref. 9. Extensive measurements of supersonic laminar, transitional, and tur-

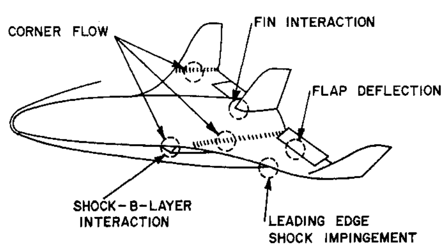


Fig. 3 Regions of strong viscous interaction on a high Mach number vehicle.

bulent boundary-layer separation over various configurations including a compression corner were made by Chapman, Kuehn, and Larsen.[10] They clearly showed the strong influence of transition on separation geometry, and the much larger pressure gradients associated with turbulent than with laminar separation.

Theoretically, the Chapman-Korst three-layer freejet model[11,12] and the integral method of Crocco-Lees[13] subsequently modified by Glick[14] reflect pioneering works toward analytically modeling the separation phenomenon. The Chapman-Korst model with its extensions and modifications by several investigators for laminar and turbulent separation has been widely applied to the base flow problem.

From practical considerations, while the main concern at lower speeds is loss of control effectiveness due to the modified pressure field caused by flow separation on a deflected flap, the dominant problem at higher speeds is the very high heat rates in the region of flow reattachment which can be as much as 100 times flat plate values. With the high Reynolds numbers of flight, flow separation will be predominantly turbulent; however, our ability to predict separated flow characteristics is almost exclusively in the laminar regime principally because the problem is more tractable as transport properties are well known.

The following sections cover briefly some experimental results and the state of theoretical prediction methods for two-dimensional laminar and turbulent separated flows related to compression corners and shock interactions.

A. Laminar Flow

The laminar boundary layer for an adiabatic or moderately cooled wall is characterized by its high susceptibility to separation. Weak shocks or small compression angles—$\Theta < 10°$—are sufficient to cause separation. Laminar separation is strongly dependent on Reynolds number, Mach number, heat and mass transfer. Experimental evidence indicates that while the point of laminar boundary-layer separation and downstream conditions are dictated by the location and strength of the disturbance—shock, compression corner—,the pressure distribution throughout the separated region is dominated by the interaction between the shear layer and external flow, and thus, is independent of the disturbance.[10,15,16] This phenomenon, termed "free interaction," forms one of the basic assumptions of theoretical analyses.[17-20]

Regarding the effect of Reynolds number, experiments on compression corners in hypersonic flow[15,16,21]—$M \simeq 6$–22—show that the separation point moves upstream with increasing Reynolds number for fully laminar flow, but reverses when flow separation becomes transitional.[15,16] The effect of high wall cooling in hypersonic flow is to markedly reduce the length of separation,[15] as shown in Fig. 4. In fact, on highly cooled ($T_w/T_0 \simeq 0.1$) compression corners attached boundary layers have been observed for compression angles as high as

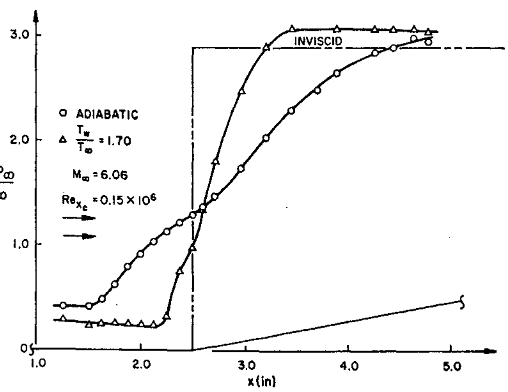


Fig. 4 Effect of cooling on laminar boundary-layer separation (from Ref. 15).

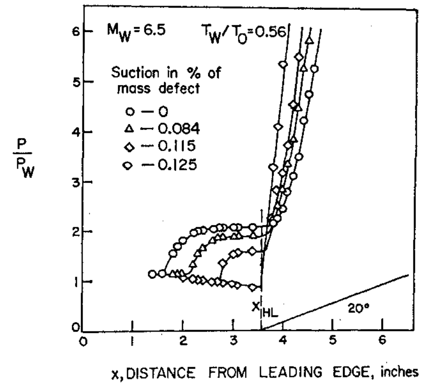


Fig. 5 Effect of mass suction on laminar boundary-layer separation (from Ref. 23).

25° (Ref. 18) and 35° (on a cylinder-flare model)[22] at Mach numbers of 15 to 20 and low Reynolds numbers; however, no adiabatic wall data are available for comparison at these conditions.

The effect of mass suction on separation due to a compression corner in hypersonic flow[23] is illustrated in Fig. 5 which shows that, by bleeding through the hinge line only 12% of the boundary layer mass defect ($\rho_e U_e \delta^*$ for an attached laminar boundary layer evaluated at the corner), fully attached flow is obtained with a compression angle of 20° for conditions whereby the incipient separation angle is only $3\frac{1}{2}°$.

End effects also influence the extent of laminar separation. If the ramp span approaches in size the separation geometry—length or height—and is bounded by sidewalls, the extent of separation is increased[15,21] due to the effect of secondary flows. The reverse is true for a configuration with a free span if the pressure of the stream beyond the side edges is lower than that on the model forebody. Similarly, if the chord of a ramp (or flap) is too small, the trailing edge flow expansion will affect the upstream location of reattachment and thus influence separation. In experiments at Mach numbers 5 to 8 with respective ramp deflection angles 10°–20°, Ball[24] finds that the region of reattachment must be at least 5–10 boundary-layer thicknesses upstream of the trailing edge for no influence to occur.

Three-dimensional effects on laminar separated flows, in the form of small streamwise vortices, have been extensively studied by Ginoux.[25] Although these vortices give rise to local spanwise heat peaks, they are not of the magnitude of flow reattachment heat rates.

From a prediction standpoint, many correlations—empirical, semiempirical, or analytic approximations—have been developed for separation pressures,[26,27] length of separation[16,24] and incipient separation conditions[16,28,29] the latter in the form $M_1\alpha_i = \lambda\bar{x}^{1/2}$, where α_i is the incipient separation angle, λ is a function of wall to freestream temperature, and $\bar{x}$ is the hypersonic viscous interaction parameter $M^3(C/Re_x)^{1/2}$.

The major advances in our ability to theoretically predict laminar flow separation characteristics came in the past half dozen years with the integral method of Lees-Reeves[17] for an adiabatic wall, followed by Holden[18] and Klineberg and Lees[19] for a nonadiabatic wall, a different integral approach by Nielsen et al.,[20] and the very promising finite-difference approach by Reyhner and Flugge-Lotz.[30] Following Lees-Reeves, Hankey and Cross[31] obtain a simple approximate closed-form solution useful for first-order engineering calculations. A critical evaluation of the three methods of Lees-Reeves-Klineberg,[17-19] Nielsen et al,[20] and Reyhner and Flugge-Lotz,[30] including comparison with each other and with experimental data was recently presented by Murphy.[32] He concludes that the methods compare surprisingly well with each other. Their agreement with the selected measured pressure distributions appear to be quite good; but heat rates

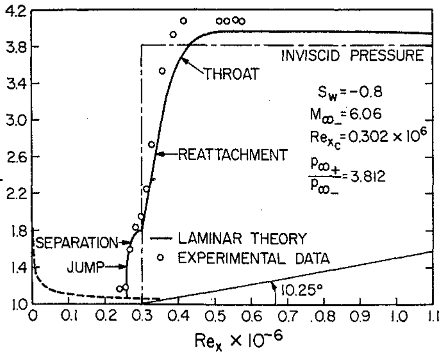


Fig. 6 Comparison of theory and experiment for nonadiabatic laminar separation (from Ref. 19).

are generally underpredicted. Although Murphy does not single out either method over the others, he does point out that the full finite-difference technique of Reyhner Flugge-Lotz makes their method the most accurate and the most general.

One can conclude that present methods are adequate to predict pressure distribution, heat transfer, and skin friction for laminar separated flow for relatively weak interactions, but further improvement is needed for strong interactions. However, an example of the present state of analytical prediction is given by a comparison of calculated and measured pressure distribution for laminar separation on a ramp illustrated in Fig. 6.

B. Turbulent Flow

The turbulent boundary layer is characterized by its ability to withstand large compressions without separating, by the insensitivity of turbulent interactions to Reynolds number, and by their rather weak dependence on heat transfer. However, their dependence on Mach number is significant. Turbulent regions of separation due to shock-boundary-layer interaction are relatively short and generally exhibit a small "knee" in the pressure distribution in contrast to the long constant pressure plateau of the laminar case, as illustrated in Fig. 7.

Experimental evidence indicates that free interaction occurs in turbulent separation up to the pressure plateau. In a comprehensive review of experimental work in supersonic flow to 1966, which includes the extensive studies of Bogdonoff,[33] Love,[34] and Chapman et al.,[10] Zukoski[35] shows that the scale for the separation geometry is the boundary-layer thickness and confirms turbulent free interaction for cases where the boundary layer is turbulent well upstream of separation. In an experimental study of turbulent interaction with a compression corner at supersonic speeds and very high Reynolds numbers, Roshko and Thomke[38] also find that, for cases of separated flow, the pressure distributions upstream of the corner are typical of a free interaction region.

The weak effect of Reynolds number on turbulent boundary-layer interactions is reflected in experimental data on incipient separation as well as on separated flow characteristics—length of separation, plateau pressure, etc. Experiments on compression corners in supersonic flow[16,36,37]—$M \sim$ 3 to 6—over a Reynolds number range of 10^6–10^7 have shown a very slight decrease in incipient separation angle with increasing Reynolds numbers. However, the more recent experiments of Roshko and Thomke[38] on a tunnel sidewall at much higher values—equivalent flat plate Reynolds numbers of 10^8 to 10^9—show the reverse trend. This reverse is explained[38,39] as the dominant effect of the wake component at high Reynolds numbers as opposed to dominance of the sublayer at lower values. Over-all variations in incipient separation angle are found to be only a few degrees for several decades in Reynolds number. However, the incipient separation angle increases with increasing Mach number as illustrated in Fig. 8. The curves are based on Reeves'[39] theory discussed further on, for $Re_{\delta*} = 10^4$. The broken curves in the figure correspond to angles required for separation lengths of 1 and 5 boundary-layer thicknesses.

The length of separation for turbulent flow appears to increase slightly with increasing Reynolds number according to measurements over the range 10^6–10^7 [16,24,36] but Roshko and Thomke's[38] measurements at higher Reynolds numbers again show a reversal of this trend. Plateau and peak pressures also exhibit little sensitivity to Reynolds number. From a large number of measurements covering two orders of magnitude in Reynolds number, Zukoski[35] shows that the plateau pressure is independent of this parameter.

Over-all, Reynolds number effects on turbulent incipient conditions or separated flow characteristics are very small at best and additional data at high Reynolds numbers and at hypersonic Mach numbers are needed for any definitive conclusion.

The effect of wall temperature, dramatic for laminar interaction, appears to be much weaker for turbulent interaction. In their extensive study of shock boundary-layer interaction at high Mach numbers, Gulbran et al.[40] find that for wall temperatures ranging from 0.3 to 0.8 of stagnation temperature, the pressure ratios for incipient separation vary from 10 to 13 at Mach 6 and 30 to 33 at Mach 8. The effect of wall temperature on a turbulent separated flow is illustrated in Fig. 9. Heat rates typical of turbulent interaction are shown in Fig. 10 for a 40° compression corner at Mach 6.5. Laminar and turbulent flat plate values are from the theories of van Driest[41] and the dashed line, representing pressure data, shows that heat transfer in the reattachment region is roughly proportional to the pressure.

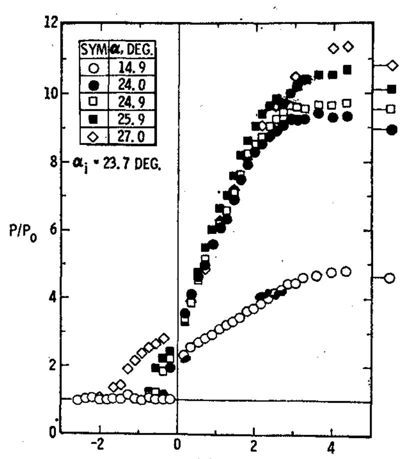


Fig. 7 Turbulent separation on a compression corner at $M = 4.92$ and $Re_\delta = 5.26 \times 10^6$ (from Ref. 38).

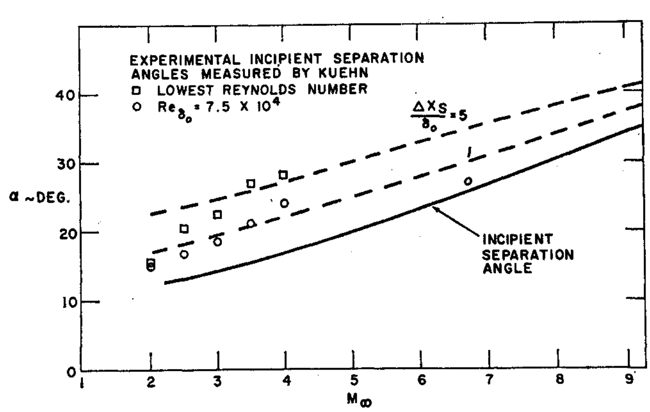


Fig. 8 Wedge angle for incipient turbulent separation (from Ref. 39).

Much of the experimental work on turbulent separation has been in the supersonic regime, primarily because of the dearth of facilities capable of yielding both high Mach numbers and high Reynolds numbers. However, the extensive studies of Watson et al.[42] and Gulbran et al.[40] of shock-boundary-layer interaction, and the compression corner investigation of Sterrett and Emery[36] and Todisco and Reeves[39] provide data on turbulent separation in the Mach 6 to 8 range. In general, few measurements exist at Reynolds numbers much greater than 10^7.

In the absence of reliable analytical methods of predicting turbulent boundary-layer interaction characteristics, one has had to rely almost entirely on empirical or semiempirical correlations for plateau and peak pressures, length of separation, and heat transfer-pressure relationships. A review of methods for predicting the pressure field for turbulent separated flow is given by Wallace.[43] Correlations for pressures and heat rates are discussed in Refs. 40 and 42. The latter work also gives a good discussion of the status of analytical methods for predicting viscous interactions.

Among early analytical studies for turbulent shock-boundary-layer interaction are Mager's[44] model for the separation pressure coefficient and pressure rise across a shock that showed good agreement with experiment for Mach numbers up to 3, Korst's[12] jet mixing theory and subsequent extensions (cf. Ref. 45) primarily related to base flows, and later Erdos and Pallone's[26] semiempirical theory for the pressure distribution which includes a weak Reynolds number dependence $Re_x^{1/10}$, and whose trend has been shown to agree with experiment but with considerable scatter.

Among recent analytical studies, Alber and Lees[46] developed an integral approach to supersonic turbulent base flows utilizing an eddy viscosity model that scales with momentum thickness, and Todisco and Reeves[39] treat turbulent separation and reattachment with an integral method utilizing a model which matches a turbulent wake-like flow to similar laminar reverse flow profiles for the separated region. A promising approach to the problem of shock wave/turbulent boundary-layer interaction, advanced by Rose, Murphy, and Watson,[47] assumes that, in the region of interaction, the boundary layer can be divided into two layers: an outer inviscid rotational layer and an inner essentially viscous layer. Rose[48] develops the concept for both laminar and turbulent interactions. The latter case involves no assumption about turbulent transport properties as the outer inviscid layer treated by the method of characteristics, is matched to an inner viscous layer which is laminar. The only empirical information required is the upstream distance to the beginning of interaction. A pressure distribution calculated by this

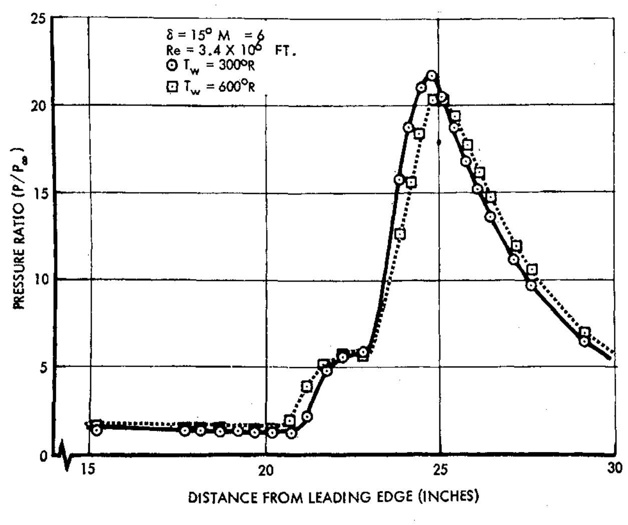


Fig. 9 Effect of wall temperature on shock-induced turbulent separation (from Ref. 40).

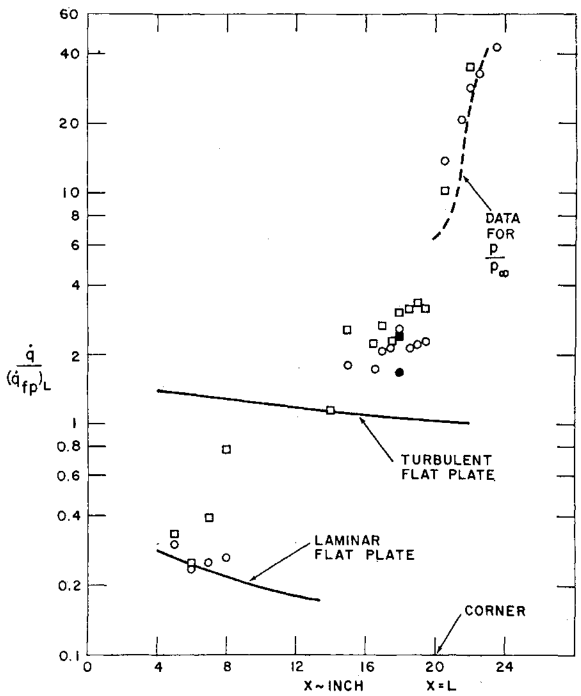


Fig. 10 Heat transfer for turbulent separation at M_∞ = 6.5 and Re_L = 1.3–1.7 × 10^7 (from Ref. 39).

method and compared with experiment is shown in Fig. 11. No comparison has yet been made for turbulent interaction heat rates.

In conclusion, knowledge of turbulent separated flow characteristics is still largely dependent on experimental data over a relatively limited range of Reynolds numbers, and prediction methods are virtually restricted to correlations derived from these data. There is a dearth of data at Reynolds numbers much greater than 10^7 particularly at high Mach numbers. Analytically, it appears that the two-layer method of Rose,[47,48] which does not require modeling of turbulent transport properties, offers the best hope for the prediction of turbulent interaction characteristics.

IV. Fin Interaction

The effect of protuberances on flat surfaces has received much attention within the past decade because of their strong influence on the pressure field and the associated regions of high local heating which result at high Mach numbers. Among early investigations, Burbank, Newlander, and Collins[49] tested some twenty configurations of surface projections both totally and partially immersed in turbulent boundary layers in supersonic flow, and made extensive measurements of surface pressures, temperatures and heat rates. For

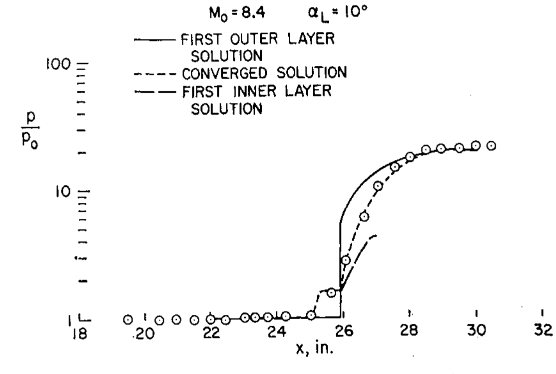
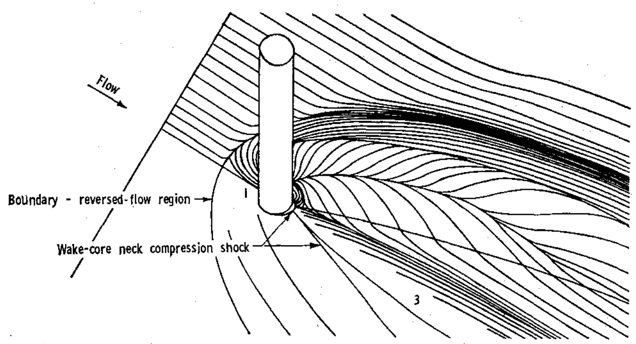


Fig. 11 Comparison of Rose's theory with experiment for turbulent shock-induced separation (from Ref. 48).



a) Cylinder (from Ref. 50)

b) Blunt fin (from Ref. 51)

Fig. 12 Plate surface flow due to blunt protuberance.

cylinders normal to the surface, they measured heat rates almost seven times flat plate values.

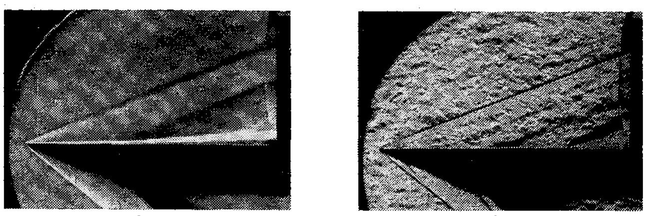
Later studies with blunt fins and cylinders mounted on flat surfaces have revealed the basic flow pattern in terms of the surface flowfield,[50–53] the upstream extent of interaction,[51,52,54,55] the surface pressure rise and fin leading edge pressures,[50–52,54–56] the effect of sweep[49,54,56–58] and surface and leading edge heat rates.[49,50,57–59]

In supersonic flow, the interaction of the strong bow shock of a blunt fin with flat plate laminar and turbulent boundary layers causes widespread upstream and lateral flow separation. The flow is characterized by a lambda shock structure in the plane of symmetry, which spreads out laterally downstream to form a broad three-dimensional interaction region with strong vortical flow. Typical surface flow patterns resulting from the interaction of a cylinder[50] and a blunt fin[51] with turbulent boundary layers in supersonic flow are shown in Fig. 12. Figure 12a was made from an oil flow photograph[50] for a cylinder mounted on the sidewall of a 4- × 4-ft wind tunnel in the Mach number range 2.5 to 4.4 and for a *Re*/ft of 3 × 10^6. In Fig. 12b the blunt fin, on a flat plate with sharp leading edge, was tested at $M = 3$, and *Re* at the flow separation line is 2.4 × 10^6. The striking similarity of the two flows tends to suggest that they may reflect the general case for turbulent interactions.

The strong resemblance of the flow structure in the plane of symmetry to that of two-dimensional interactions as, for instance, ahead of forward facing steps, is revealed by several investigations and discussed below. One might conjecture that the former is very closely analogous to two-dimensional separation with mass suction, as part of the oncoming flow (at least part of the boundary-layer mass flow) is scavenged and carried away laterally by a horseshoe vortex structure before it reaches the fin.

The separation distance upstream of a fin leading edge l_{sep} is found to scale with fin diameter, d, for conditions whereby the fin is slender and its diameter is larger than the boundary layer thickness.[51] The separation distance is considerably greater for laminar than for turbulent flow[51] as illustrated in Fig. 13. For laminar flow it is Reynolds number dependent,[51] and limited data indicates a general increase in length with increasing Reynolds number as is the case for two-dimensional interactions. For turbulent flow there is extensive experimental evidence that the separation distance, l_{sep}/d, is insensitive to Reynolds number. Young et al.[51] and Voitenko et al.[52] find $l_{sep}/d \simeq 2$ in experiments with fins at $M = 3$ and cylinders at $M = 2.5$, respectively, covering a Reynolds number range from 2.4 to 18.5 × 10^6. Price and Stallings[54] find some variation from l_{sep}/d somewhat less than 2 to almost 3 at $M = 2.3$ to 4.4; however their fin diameters are only about $\frac{1}{10}$ to $\frac{1}{2}$ of the boundary-layer thickness. Westkaemper,[55] based on his own measurements and those of other sources over a wide range of Reynolds numbers, finds a somewhat higher value, $l_{sep}/d \simeq 2.6$, but virtually independent of Mach number over the wide range from $M = 2$ to 21. He further shows that the length of a cylinder does not influence the separation distance providing it is larger than the diameter (and the diameter larger than the boundary-layer thickness, presumably); however, a sharp decrease occurs for shorter cylinders.

The centerline pressure distribution reflects a rise to a plateau or peak, followed by a large dip, and then a sharp rise to a second peak at the foot of the fin for both laminar[51] and turbulent flow[51,52,54] as illustrated in Fig. 14. It is found that centerline pressure distributions for both laminar and turbulent interactions bear a strong resemblance to those for two-dimensional separation; however, the plateau or peak values are consistently slightly lower.[51,52,54] Comparisons made of plateau pressures with two-dimensional correlations,[51] and of the pressure rise for turbulent flow with two-dimensional data,[52] strongly suggest that "free interaction" exists for these highly three-dimensional flows. According to Ref. 51, estimates of the pressure rise can be obtained from such two-dimensional empirical correlations as that of Hill[27] for the laminar case and Sterrett and Emery[36] for the turbulent case. An approximate analytical expression developed by Truitt[60] for the turbulent pressure rise across the separation shock, includes a weak Reynolds number dependence and requires an empirical constant. It has only been compared with limited data and underpredicts the measurements of Ref. 51.



a) $M_1 \approx 3$, $l/d = 7.80$ and $Re_{sep} = 101{,}000$

b) $M_1 \approx 3$, $l/d = 7.80$ and $Re_{sep} = 3{,}500{,}000$

Fig. 13 Schlieren photographs of laminar and turbulent fin interactions (from Ref. 51).

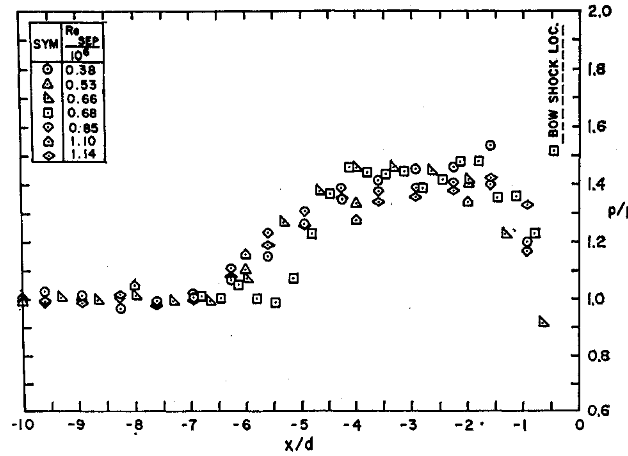


Fig. 14 Centerline pressure distribution for laminar fin interaction at $M = 5$ (from Ref. 51).

Pressure distributions along the leading edge of fin or cylinder reflect a rise above stagnation point values below the triple-point of the lamda shock followed by a sharp drop-off at the foot,[51,55] as illustrated in Fig. 15. This is characteristic of the higher pressure recovery following compression through oblique shocks. Westkaemper[56] utilizes Truitt's[60] pressure rise relationship for turbulent interaction to estimate the height of the triple point and the peak pressure on the leading edge. He attributes the fact that measured values are significantly less than the calculated peak pressure, to three-dimensional effects not accounted for in the analysis.

The effect of leading edge sweep on the extent of interaction has been investigated by Price and Stallings[54] for supersonic turbulent flow, and by Thomas[56] for a fin on a blunted flat plate in hypersonic flow at low Reynolds numbers. Price and Stallings[54] find that, for sweep angles from 0° to 30° there is a sharp decrease in extent of separation but no change in the first peak pressure; but, for larger sweep angles, both decrease until they are barely perceptible at 75°. Investigations of heat transfer and pressure on swept cylinders mounted on wedges have been made by Beckwith[57] for 20° and 60° sweep at $M = 4.15$, and Bushnell[58] for 45° and 60° sweep at $M = 8$. They find that the higher pressures and heat rates measured near the base of the cylinders in the region influenced by the wedge can be reasonably well predicted by infinite swept cylinder theories providing local flow conditions are used. Further detailed discussion of heating on blunt leading edges is given in the section on Shock Impingement.

Local regions of high heating resulting from the fin interaction on a plate or wedge surface, appear near the base and extend downstream at an angle to the fin as shown in Fig. 16. Note the similarity with Fig. 1. These high heat rates appear to be associated with a reattachment line, or the flow impingement[50] side of a horseshoe vortex—cf. line 2 on Fig. 12a.

Studies of the interaction of wedge-shaped fins with sharp leading edges on flat surfaces including surface pressures, heat rates, and prediction correlations are reported by Neumann and Burke[5] and in the extensive investigation of Kaufman et al.[3] Away from the fin, the interaction is essentially that of a swept-planar shock with a boundary layer as studied theoretically by Martellucci and Libby.[61]

In conclusion, surface flow characteristics due to the interaction of a blunt fin with a flat surface are known at least qualitatively; however, details of the flowfield including the strong horseshoe vortices and their scavenging action on the oncoming stream are far from well understood. Surface pressure distributions in the plane of symmetry closely resemble those for two-dimensional flow separation for both laminar and turbulent flow, but the peak or plateau values are somewhat lower. The turbulent case appears to be independent of Reynolds number and the separation distance is found to be 2–2.5 fin diameters upstream of its leading edge. Zones of high heating spread laterally downstream from the fin leading edge. There are presently no theoretical means of prediction of fin interaction characteristics.

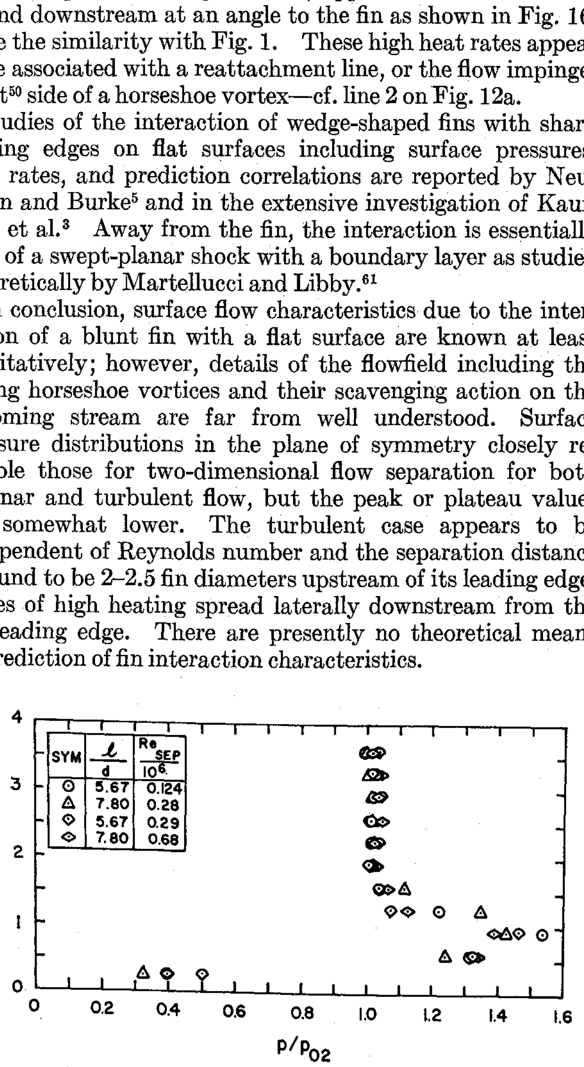


Fig. 15 Pressure distribution along fin leading edge at $M = 5$ (from Ref. 51).

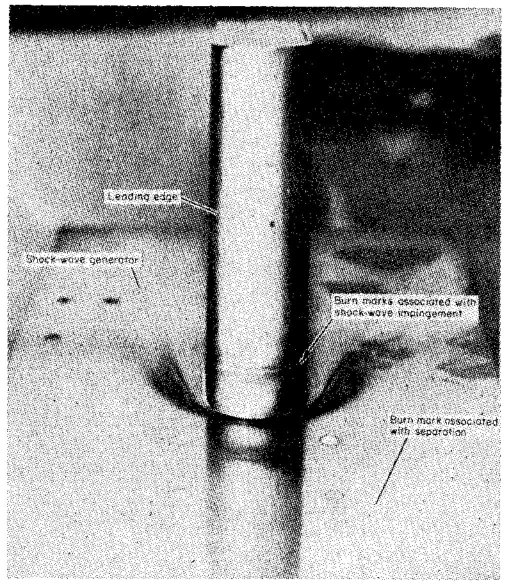


Fig. 16 Regions of high heating due to fin interaction at $M = 14$ (from Ref. 59).

V. Axial Corner Flow

Supersonic flow in axial corners formed by two intersecting flat surfaces has been the subject of experimental investigations for slightly over a decade. A corner interaction occurs as a result of initial flow compression on one or both surfaces due either to boundary-layer displacement effects[62–66] or to the inclination of the surfaces with respect to the freestream[4,67–69] or both.

In spite of the inherent simplicity of the corner geometry, the flowfield is quite complex and was virtually unknown until the recent detailed experimental investigation of Charwat and Redekopp[67] on intersecting wedges. Prior investigations were concerned primarily with surface conditions.

Early investigations of 90° corners[62–64,66] made up of flat plates aligned with high Mach number streams revealed higher pressures, and heat rates of several times flat plate values in the corner region. Cresci's[66] results at low Reynolds numbers suggest that the main parameter reflecting the magnitude of the interaction when boundary-layer displacement effects are dominant, is the classical hypersonic viscous interaction parameter $\bar{\chi} = M^3 (C/Re_x)^{1/2}$. His peak pressures in the corner region for values of $\bar{\chi}$ from 1 to 10, appear to be roughly 1.5 times the two-dimensional interaction pressures.

Recognizing the importance of the inviscid flow structure, Charwat and Redekopp[67] made extensive Pitot pressure as well as surface measurements in corners of intersection wedges of various angles—symmetrical and asymmetrical configurations—over the Mach number range from 2.5 to 4. The basically conical flow structure representative of their findings and illustrated in Fig. 17a, shows a break in the individual wedge shocks which are joined by a third shock, two slightly curved shocks from the outer shock interactions to the wedge surfaces (between Zones II and III), slip surfaces from the same intersection points toward the corner intersection (between Zone II and I), and a spread of the corner disturbance well outside of the inner shocks as reflected by Zone III, beyond which the flow is essentially two-dimensional (Zone IV). Subsequent investigations with intersecting wedges in hypersonic flow[4,65,69] have revealed the same basic flow features, as illustrated in the photograph of Fig. 17b taken at the NASA Langley Center with their sweeping electron beam technique,[4,69] and a Mach number of 19. The dark region on the horizontal surface is the boundary layer. Thus there is growing evidence that the shock structure and flow pattern found by Charwat and Redekopp may well reflect the general case

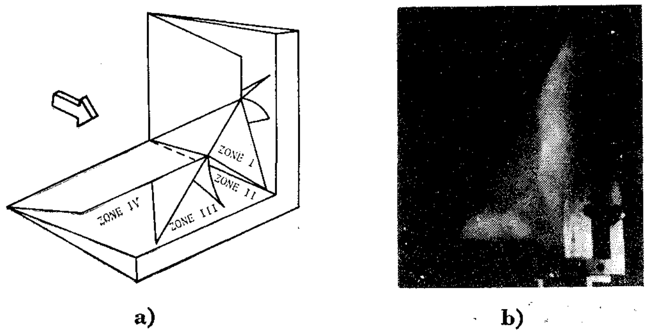


Fig. 17 Corner flow structure (from Ref. 4): a) flowfield of Charwat and Redekopp at $M = 3.17$; b) NASA electron beam flow visualization at $M = 19$.

for axial corners in high Mach number streams. Some differences with other experiments are noted, such as the fact that Charwat's inner shocks curve outward whereas those of Watson[4,69] curve inward, as pointed out by Bertram[4]; also, the slip surfaces in Ref. 67 are shown to join at the wedge intersection (see Fig. 17a) whereas they merge before they reach this line in the work of Ref. 69. Theoretical arguments[70] favor the latter case. These differences in detail will probably be clarified in future studies now that the basic corner flow structure is known through Charwat and Redekopp's pioneering work.

The corner problem may be viewed as one involving a strongly disturbed inviscid flowfield, and, in turn, the interaction of this flowfield with surface boundary layers. A surface flow pattern in the corner region of intersecting wedges is illustrated in the oil flow photograph of Fig. 18. It clearly shows that the flow is essentially conical except near the leading edge where boundary-layer displacement effects are large—values of $\bar{\chi}$ greater than 4 for the configuration shown.[69] The inner S-shaped line reflects a vortex (also apparent at the right end of the dark surface region in Fig. 17b) as noted by Bertram[4] and also found by Stainback[68] in his studies. The outer region of influence spreads far from the corner, well beyond the inner shock location. Based on his experiments, Charwat suggests that this broad region is not the extent of interaction of the inner shock with the boundary layer, but is rather an essentially inviscid compression region. However, the outer oil accumulation line of Watson's study[69] strongly suggests flow separation, as further discussed below. It is of interest to note features of similarity between Fig. 18 and the surface flow for the blunt fin interaction (turbulent in this case) shown in Fig. 12b.

Representative spanwise pressure and heat-transfer distributions in a corner of intersecting wedges from Charwat and Redekopp's[67] data at $M = 3.17$ and wedge angles of 12.2°, and Watson's data[69] at $M = 20.3$ and 10° angles are compared by Bertram and Henderson[4] in Fig. 19. Both pressure distributions exhibit the sharp rise and corner plateau associated with the inner shock, following a smaller pressure rise over a broad span as reflected by Zone III in Fig. 17 and by the outer region of Fig. 18. The heat-transfer distributions exhibit a marked dip or trough to values well below those for a two-dimensional wedge, followed by a sharp rise to two distinct peaks the second of which is ten times the wedge value for the Mach 20 case. The peaks have been associated with the strong vortex or vortices adjacent to the corner;[4,68,69] however, the outboard trough which also appeared in some of Cresci's and Stainback's measurements has remained unexplained. While flow separation due to impingement of the inner corner shock has been mentioned in the literature,[68,69] the extent of separation in relation to surface measurements has not been fully discussed.

The corner heat-transfer distribution has distinct features in common with two-dimensional shock wave laminar boundary-layer interaction—a drop in heat-transfer rates beyond separation (the trough) followed by a rise to high values (peaks) at reattachment. The pressure distribution also exhibits common features—an initial rise to a plateau followed by a sharp peak. From these strong features of similarity coupled with the oil flow pattern, the present author suggests that the corner flow boundary layer separates at the outer accumulation line shown in Fig. 18 and corresponding to Zone III in Fig. 17. The drop in heat transfer between the two peaks, coupled with the oil flow pattern, also suggests a region of outflow from the surface, and hence a possible system of three vortices—a weak outer one, and two strong ones inboard. It is conceivable that much of the original boundary-layer mass flow is scavenged and carried downstream by these vortices as appears to be the case for the blunt fin interaction. Further investigations with turbulent boundary layers, for which separated regions are considerably smaller, would help clarify the nature of the flow.

The sharp drop in heat transfer at the intersection of the surfaces, also clearly shown by Stainback[68] and Cresci,[66] is

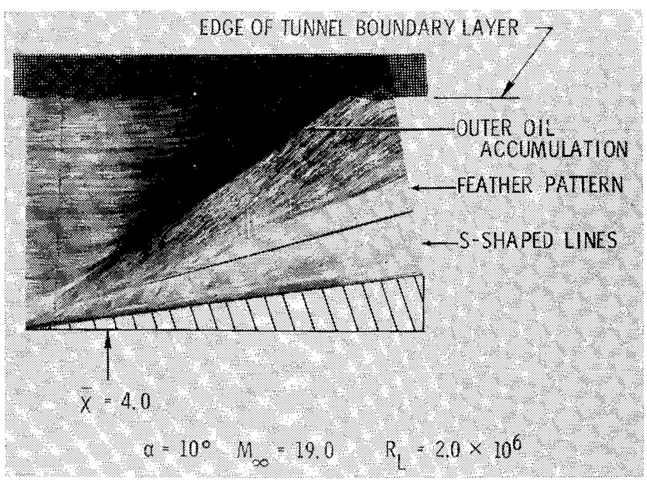


Fig. 18 Oil flow pattern in corner region (from Ref. 69).

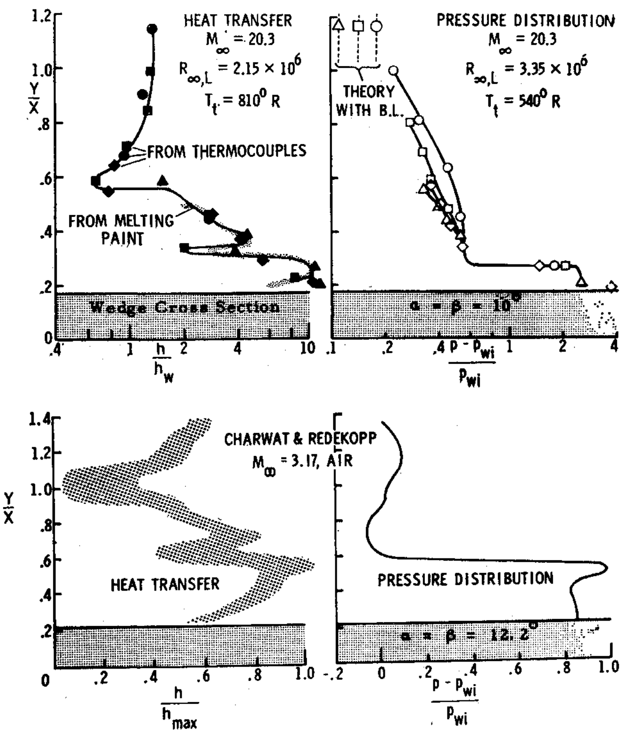


Fig. 19 Spanwise heat-transfer and pressure distributions for $M = 3.17$ and $M = 20.3$ (from Ref. 4).

attributed to the mutual interaction of the surface shear flows,[68] and theoretically predicted by Rubin.[65]

Although the discussion above deals with 90° corners, studies have been made with corner angles varying from 60° to 270° (Refs. 66, 68, and 69). They show in general higher pressures and heat rates, and an outer displacement of the inner shock, with decreasing corner angle.

Correlations of the value and location of laminar peak heating in a corner region are given by Stainback[68] based on his extensive studies at a Mach number of 8, and illustrated in Fig. 20 for a symmetrical wedge configuration with angles of 5° and 10°.

Virtually all experimental work on corner flow involves laminar boundary layers; however, practical cases associated with hypersonic cruise vehicles will most probably involve turbulent flow.[68] Some preliminary turbulent results were obtained by Stainback[68] for a corner interaction of sharp-edged plates aligned with the flow, and for the intersection of a blunt plate with a sharp-edged one. The former model shows no measurable change in heat transfer in the corner region; however, in view of the turbulent boundary layers and small associated displacement effects, the bow shocks and, therefore, the corner interaction were undoubtedly very weak. The latter model reflected a rise in heat transfer, but the peak in relation to flat plate values was not as high for the turbulent as for the laminar case, as shown in Fig. 21.

Theoretical work descriptive of supersonic flow in an axial corner is virtually nonexistent. Early studies of the inviscid flowfield were either linearized or assumed intersecting bow waves, and studies of boundary layers in an axial corner assumed a uniform stream, neither of which are representative of the supersonic interaction as found experimentally. These studies are noted, for example, in the extensive reviews of Ref. 7 and 70, and also in Refs. 66 and 67.

Goebel[70] considers a theoretical model of the inviscid flow in a corner based on the experimental findings of Ref. 67. Utilizing an iterative procedure which requires matching of pressures and flow direction across the slip surfaces (separating Zones I and II in Fig. 17a), he determines the triple-shock line and the central flow region (Zone I) which is shown to collapse above the line of intersection of the wedges, unlike the results of Ref. 67 but in agreement with those of Ref. 69. Goebel's analysis does not provide a description of the inner and outer flow regions nor does it yield a unique solution for the inner shocks.

A theoretical merged layer analysis developed by Rubin, based on a single set of equations valid throughout a viscous and inviscid flowfield was applied to the corner configuration and shown to give fairly good agreement with measured surface pressures, heat transfer, and skin friction at high Mach numbers and low Reynolds numbers[65] whereby the shear layer fills almost the entirety of the disturbed region.

For the high Reynolds number corner problem associated with hypersonic cruise vehicles, there appears to be no analysis presently available for even the inviscid flowfield and its embedded shock structure. Regarding viscous effects, the only analytical study known to the author is that of Martellucci and Libby[61] for the interaction of the bow shock of a wedge normal to a flat plate, with the laminar boundary layer on the plate.

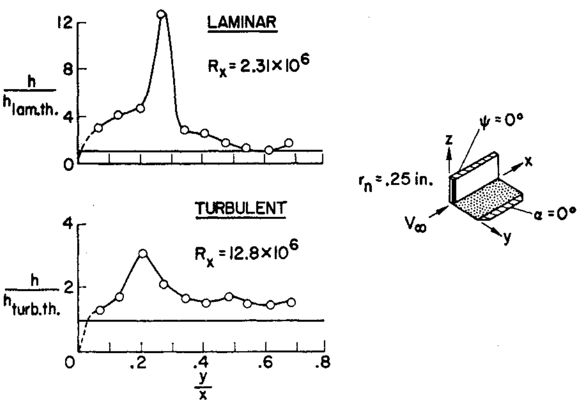


Fig. 21 Comparison of laminar and turbulent heating in a corner at $M = 8$ (from Ref. 68).

Having discussed the complex structure of flows in corners, it is of interest to point out examples of corners with swept edges which have simple inviscid solutions: the "caret" wing which has a uniform (wedge) flow with a single plane shock joining the edges, and the Gonor configuration illustrated in Fig. 22 (cf. p. 223 of Ref. 71), which may be viewed as an off-design caret wing, wherein the bow shocks intersect and their reflected (inner) shocks terminate at the wedge surface without further reflection, by a suitable cutout in the corner thus accommodating the inner surface to the new flow direction.

In conclusion, the flow structure in an axial corner formed by two compression surfaces in a supersonic stream is qualitatively known for laminar flow and includes strong vortices and embedded shocks. Zones of high heating and high pressures extend along near-conical rays from the corner apex. Details of the vortex structure and the extent of flow separation on the surfaces need to be further explored. Also, there is virtually no data on corner interactions with turbulent boundary layers.

Although a theoretical merged layer analysis shows promise of predicting corner flow characteristics reasonably well for relatively low-density flows, there exists no adequate method

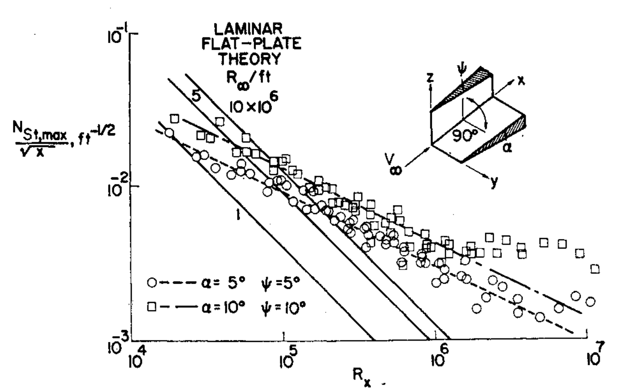


Fig. 20 Laminar peak heating in corner region for $M = 8$ (from Ref. 68).

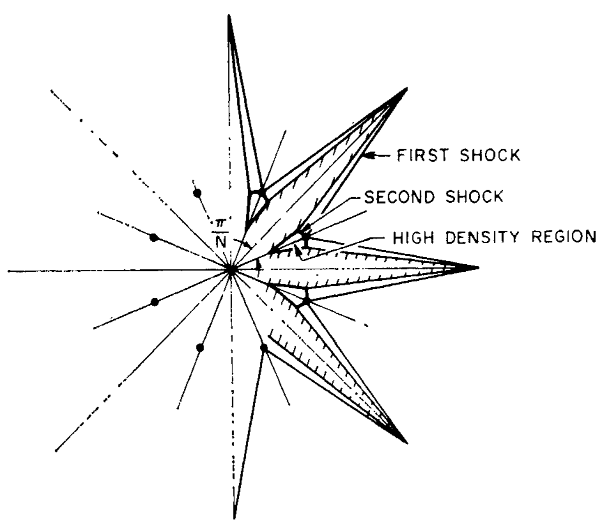


Fig. 22 Gonor's corner configuration (from Ref. 71—reprinted by permission of Academic Press, copyright 1966).

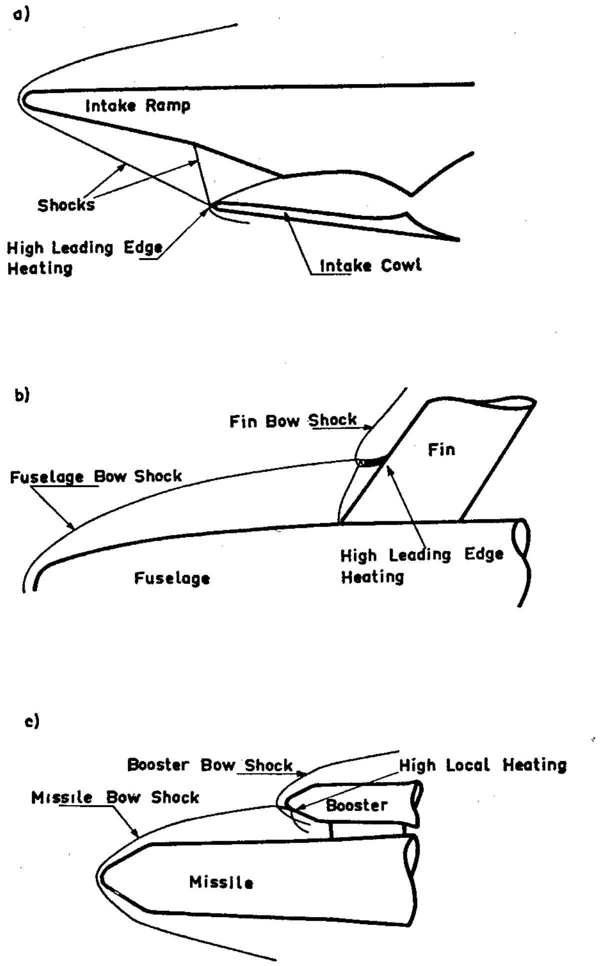


Fig. 23 Practical examples of shock impingement heating (from Ref. 76).

of predicting even the inviscid flow structure for the high-density corner flows likely to be encountered on a high Mach number vehicle.

VI. Shock Impingement

The most severe aerodynamic heating is undoubtedly due to shock impingement or the interaction of an externally generated shock with the bow wave at the leading edge of a blunt body. Examples of practical configurations in which shock impingement arises are shown in Fig. 23.

Since Newlander's[72] study of shock impingement on a right circular cylinder a decade ago, only a limited number of investigations of the problem over the range of Mach numbers from 2 to 19 are found in the literature. Most of these investigations utilize circular cylinders or blunt fins mounted on wedge or plate shock generator[57–59,73,74] in which case a second impinging shock may be due to boundary-layer separation resulting from the fin-plate interaction (see Sec. IV). Measurements made were primarily of surface conditions—pressures, heat rates, oil flow, temperature sensitive paints.

Leading edge sweep was found early to have a marked influence on shock impingement heating. For highly swept cylinders or fins—near 45° or more, no local increases in heating are observed[57–59,73] and leading edge heat rates are found to be reasonably well predicted by simple infinite swept cylinder theories using local flow conditions.[57,59,75] For unswept or moderately swept cylinders, local high heating is noted at the leading edge in the vicinity of the intersection of the external shock with the cylinder bow wave.[59,72,73,75] Heat peaks as high as ten times stagnation values have been observed,[59,73] as shown in Fig. 24. The very localized nature of the heat peak due to shock impingement is illustrated by the narrow burn mark on the fin in Fig. 16 and the pointed dark strips (melted paint) at the outboard wing leading edge of the NASA MSC orbiter model shown in Fig. 25. This figure reflects offdesign conditions of the orbiter; the heat peaks do not appear at the 60° angle of attack of the re-entry configuration.

The apparent anomaly in leading edge heating between unswept or moderatly swept configurations, and highly swept ones is, however, easily explained. The heat peak has been associated with the impingement of a shear layer originating at the intersection of the external shock and the bow shock of the blunt fin.[59,72,75] For high sweep angles the shear layer does not impinge on the leading edge, but rather, flows tangent to it,[75,76] and therefore no heat peak is observed. This explanation is basically correct; however, the flow structure for low sweep angles is considerably more complex than that of a simple shear layer as will be discussed further on.

While early investigators identified the problem of shock impingement and contributed much needed data on local heat rates, it was not until the extensive study of Edney,[76] who obtained very detailed schlieren photographs of a remarkable quality, that understanding of the interference flowfield was gained. Edney measured shock impingement heating on a hemisphere, a blunted cone, and a flat-faced cylinder at Mach numbers of 4.6 and 7. Using the quasi-steady technique of laterally sweeping the models relatively slowly across a wedge-generated shock in the test section, he obtained accurate local measurements of heat rates and pressures as as well as schlieren visualization. For a model such as the hemisphere, a single test covers the whole spectrum from high positive sweep angles—the angle of sweep and the impinging shock have the same orientation—through zero sweep, to high negative sweep angles. The locus of peak heat rates as the impinging shock, produced by a 5° wedge, moves across the hemisphere is shown in Fig. 26 for a Mach number of 7. Also shown is the heat-transfer distribution in undisturbed flow. In the vicinity of the stagnation point of the sphere the shock impingement point and the corresponding heat peak are virtually coincident. Away from the stagnation point, the peak is somewhat downstream of the corresponding shock impingement location. Note that the peak heat-transfer distribution is not symmetrical. The highest peak occurs away from the stagnation point, corresponding to slightly negative sweep. The heat-transfer distribution for a fixed location of the impinging shock would have a sharp rise to the peak similar to that shown in Fig. 24. Figure 26 also shows that heat rates for high positive sweep are only slightly affected by shock impingement, as found by other investigators.[57–59,73]

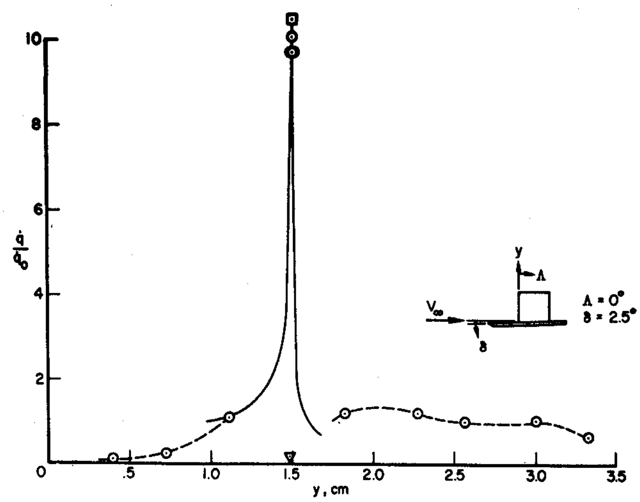


Fig. 24 Effect of shock impingement on fin leading edge heat transfer at M = 14 (from Ref. 59).

Peak pressures follow very nearly the same distributions as peak heat rates, with maximum values at essentially the same location away from the stagnation point of the hemisphere. This fact suggests the validity of pressure—heat-transfer correlations.[76] Edney finds that peak heating increases with increasing impinging shock strength and Mach number.

From his detailed schlieren photographs, Edney identifies six different types of interference flowfields associated with shock impingement, dependent on whether the flow in the shock layer is entirely subsonic (unswept or slightly swept edge), slightly supersonic but with subsonic regions (moderately swept), or entirely supersonic (highly swept), and also on whether sweep is positive or negative with respect to the inclination of the impinging shock. Three types of interference flowfields associated with positive sweep, the most likely to be encountered on vehicle wings and vertical control surfaces, are shown in Fig. 27. Figure 27a shows a typical interference structure for an unswept configuration, with a break in the bow shock and a supersonic jet in the otherwise subsonic shock layer, which impinges on the leading edge. The supersonic jet, which includes embedded shocks, is bounded by shear layers and its exact structure is dependent on the bow shock detachment distance in relation to viscous effects, i.e., some Reynolds number. Because the shock layer flow is subsonic, the structure of the bow shock in the neighborhood of its intersection with the external shock, is not readily predictable. Thus, this type of interaction which results in the highest heat rates, is very likely the most complex and least understood of any encountered in practical applications. Figure 17b shows a moderately swept configuration for which the shock layer flow is slightly supersonic. The jet has thinned considerably and a shear layer originates at the intersection of the bow wave with an embedded shock behind which the flow is locally subsonic. Edney points out that jet and shear layer may strike the leading edge far downstream of the impingement point, where, because of diffusion, their influence on heat rates will be considerably lesser than for the unswept case. The embedded shock impinging on the leading edge may cause a local separation and boundary-layer transition as suggested by Edney. In Fig. 27c, reflecting a highly swept edge, the shock layer flow is entirely supersonic and the shear layer misses the surface, hence high heat rates do not arise. It is for this configuration that simple infinite swept cylinder theories have been found to adequately predict heat rates when local flow conditions are used.[57,59,75]

From the various investigations, parameters besides angle of sweep, which influence shock impingement heating are: the geometry of the leading edge including possible tip effects,[75] the shock detachment distance,[75,76] the strength of the impinging shock,[76] the Mach number,[76] the nature of the boundary layer on the leading edge,[75,76] and real gas effects.[76]

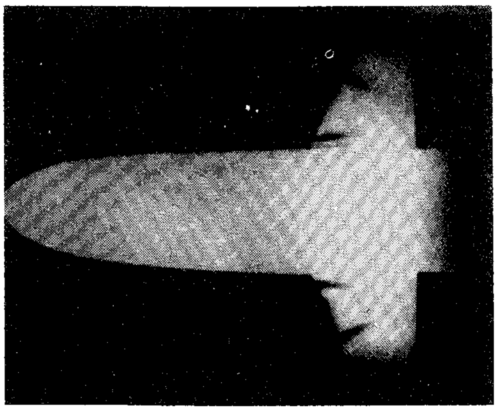


Fig. 25 Thermal paint indication of shock impingement heating on NASA MSC orbiter model for off-design conditions: $\alpha = 40°$ at $M = 8$ (courtesy of NASA).

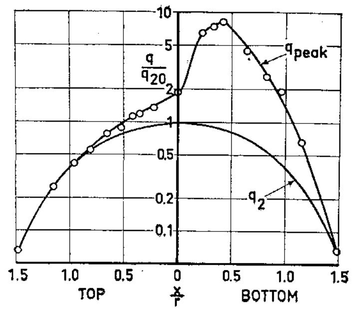


Fig. 26 Locus of peak heating on hemisphere with varying shock impingement point at $M = 7$, $\Theta_w = 5°$ (from Ref. 76).

Regarding the latter, Edney shows a large effect of γ (the ratio of specific heats) on peak heating, with as much as one order of magnitude difference between $\gamma = 1.2$ and 1.67.

A few theoretical models for shock impingement have been advanced which Edney refers to as acoustic disturbance. shock-induced vorticity, and divided flow models. Although some reflect correct features of the flow, they are generally inadequate for practical purposes, and therefore are not discussed herein. Detailed discussion of these theories can be found in Ref. 76. Based on his observations, Edney is able to calculate the inviscid flow structure in the vicinity of shock interaction for some interference patterns. He also develops a simple expression for peak heating for normal impact of a jet which gives good agreement with his experiments but requires the jet dimensions, and shows that peak heating increases as the square root of the pressure.

In conclusion, it is found that shock impingement can result in sharp heat peaks as high as ten times stagnation values on unswept or slightly swept surfaces, that the peaks decrease with increasing sweep angle and disappear for highly swept edges. Heat rates for the latter are predictable by infinite swept cylinder theories. The qualitative structure of the interference flowfield and its variation with sweep has been determined and shown to be extremely complex. More experiments such as Edney's are needed for quantitative measurements on various configurations, particularly blunt

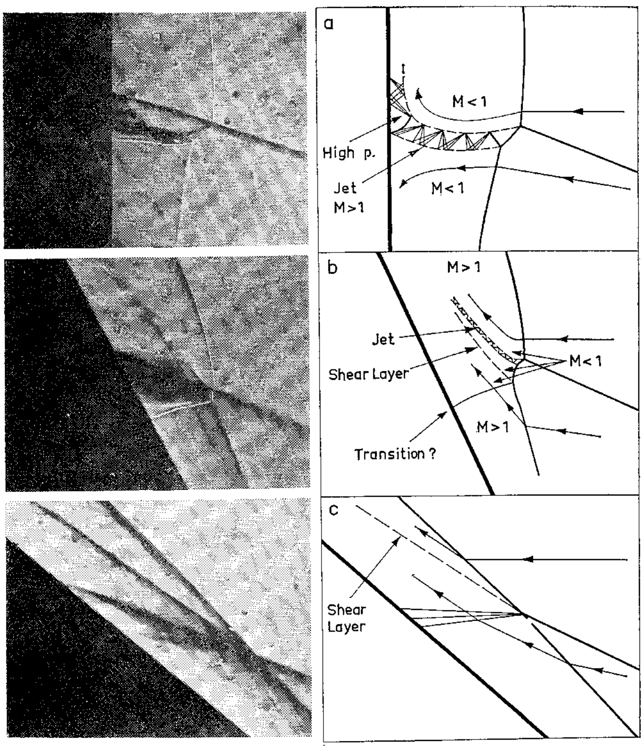


Fig. 27 Effect of varying sweep angle of cylindrical fin on flow interference pattern at $M = 4.6$, $\Theta_w = 5°$ (from Ref. 76).

fins as they reflect wings and control surfaces and blunt configurations representative of cowl lips and booster-orbiter combinations. Theoretically there exists no adequate model to predict shock impingement heat rates.

VII. Summary and Conclusions

High Mach number cruise and lifting re-entry vehicle configurations include many regions of high compression that can cause boundary-layer separation and reattachment where very high local heating arises. Such regions include deflected flaps, shock interactions in inlets, blunt fins or other protuberances on fuselage or wing, axial corners in inlets, wing-body and fin-wing junctions, and such regions at the leading edge of wings and fins which are subjected to the impingement of shocks from the fuselage or other forward component.

Basic studies related to these interactions include flow separation on compression corners and due to shock-boundary layer interaction, and the three-dimensional problems of the interaction of a blunt fin on a surface, flow in an axial corner, and leading edge shock impingement.

With respect to two-dimensional interactions, present theoretical methods are adequate to predict pressure distribution, heat transfer, and skin friction for laminar separated flow for relatively weak interactions, but further improvement is needed for strong interactions. For turbulent separated flows, prediction methods are still largely restricted to correlations based on data over a relatively limited Reynolds number range. There is a dearth of data at Reynolds numbers much greater than 10^7 particularly at high Mach numbers. Reattachment heat rates as much as 100 times flat plate values have been measured. A promising analytical method for the prediction of turbulent interaction characteristics utilizes a two-layer approach which matches an inviscid, rotational outer layer to an inner viscous layer which is laminar (sublayer-like).

The three-dimensional interactions involve highly complex flowfields with strong vortices and internal shocks. For the blunt fin interaction, surface flow characteristics are known at least qualitatively; however, details of the flowfield including the structure of the strong horseshoe vortices are far from well understood. Results to date indicate that the turbulent interaction may be independent of Reynolds number. Zones of high heating on the plate surface are found to spread out laterally and downstream from the fin leading edge.

In an axial corner in a supersonic stream, the flow structure is qualitatively known for laminar flow; however, there is virtually no data for turbulent flow. Zones of high heating and high pressure extend along near-conical rays from the corner apex. Heat rates one order of magnitude higher than flat plate values have been measured for laminar flow. Preliminary indications are that the rise in heating is lesser for turbulent than for laminar flow; however, the absolute values may be higher.

Shock impingement heat rates as high as ten times stagnation values have been measured on unswept or slightly swept leading edges. These heat peaks decrease with increasing sweep angle and vanish for highly swept edges for which heat rates are found to be well predicted by infinite swept cylinder theories. The structure of the interference flowfield due to shock impingement is qualitatively known and shown to be extremely complex as it includes embedded supersonic jets and shocks.

Although a few simple analytical models have been advanced for local characteristics of some of the three-dimensional interactions, none are adequate for the prediction of the location and value of peak heat rates; furthermore, there exist no adequate theoretical models to calculate even the inviscid flow structure of these interactions with their embedded shocks, corresponding to the high-density conditions of flight in the sensible atmosphere.

In conclusion, improvements in the theoretical modeling of two-dimensional separation and more experimental work on the flow structure as well as surface conditions for three-dimensional interactions are needed. In general, experimental studies of both two- and three-dimensional interactions at high Reynolds numbers ($\sim 10^8$) and high Mach numbers representative of conditions encountered in flight need to be made.

In the final analysis, the severity of heating due to viscous interactions may be such that it is far more desirable to design around them than attempt to cope with them through weighty replaceable ablation shield or equally bulky and sophisticated forms of cooling. Whichever the practical solution, it is essential to fully understand the characteristics of viscous interactions.

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Theoretical Analysis of Vortex Shedding from Bodies of Revolution in Coning Motion

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A theoretical flow model for the steady asymmetric vortex system shed from a slender body in coning motion is described. The model was developed using potential flow methods and slender-body theory, and provides for the calculation of the strengths and positions of two unequal concentrated vortices and the resulting force distribution induced on the body. The vortex motion is determined in the flowfield which consists of a portion described by a velocity potential plus a portion due to rotation. The method of determining the initial conditions for the vortex motions is discussed. Comparisons are made between predicted and experimental values of side forces and side moments for slender cones and ogive-cylinder combinations in lunar coning motion.

Introduction

THE nature of vortex formation on an inclined slender body and its relation to the two-dimensional flow over a cylinder was recognized some 20 years ago.[1] Since then a considerable amount of effort has been expended in studying the nature of the flow over such bodies and the vortex-induced force distribution on them.[2,3] It has been only recently, however, that the presence of a steady vortex pair on a slender body in a coning motion has been established.[4] The work described in this paper is a theoretical analysis of that problem. The work was reported in detail in Ref. 5.

The coning problem is of importance for spinning bodies which encounter a pitch-roll resonance condition, from which a lunar coning motion (roll lock-in) can develop with unacceptably high angles of attack. The reasons for the development of this type of motion are not well understood. Tobak[4] has developed a formulation for the aerodynamic moment system in lunar motion which does not depend on constructing the nonplanar motion as the sum of two planar motions. This approach permits coupling of the two planar motions and identifies two types of "Magnus moments": one due to spin about the body axis and one due to rotation of the angle of attack plane (the plane formed by the wind vector and the body axis). The existence of vortices over the body provides a potential source of coupling and nonlinear moments of the latter type.

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